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Multi-variant Optimization in Semiconductor Heat Sink Design

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ABSTRACT

There are a number of traditional techniques applied to the optimization of heat transfer devices such as heat sink enhanced surfaces or heat exchangers. There are, however, no methods or mathematical studies devoted to optimization of hierarchical heat transfer devices. Design optimization procedures for transport in porous structures and enhanced heat transfer surfaces are formulated and developed in this study for the purpose of hierarchical heat sink design. Mathematical formulation of a hypothetical heat transfer surface with a priori unknown heat transfer enhancing elements is developed using a two scale description based on volume averaging theory (VAT). Second order turbulent model equation sets based on VAT are used to determine turbulent transport and two temperature diffusion in a non-isotropic porous media and inter-phase exchange at a rough wall. Though several different closure models for the source terms for spacial uniform, non-uniform, non-isotropic highly porous layers have been successfully developed, quite different situations arise when attempting to describe processes occurring in irregular, random or even unknown morphologies. After simplification by assuming regularity of the spacial morphology, this problem is still has a large number of optimization space dimensions. In a laminar heat transfer region, the problem is 6 to 8-D and in turbulent it is 8 to 9-D. A few methods and statistical techniques used for problem specification are discussed.

Nomenclature

α_f	-	thermal diffusivity $[m^2/s]$
c_d	-	mean drag resistance coefficient in the REV [-]
C_d	-	drag coefficient [-]
c_p	-	specific heat $[J/(kg \cdot K)]$
d_h	-	hydraulic dynameter, $[m]$
d_{por}	-	characteristic length of porous medium $[m]$
d_p	-	side length or diameter of the unit $[m]$
dS	-	interphase differential area in porous medium $[m^2]$
E	-	Effective [-]
∂S_w	-	internal surface in the REV $[m^2]$

f	-	friction factor
$\widetilde{f}$	-	averaged over $\Delta\Omega_f$ value f
$< f >_f$	-	value f, averaged over $\Delta\Omega_f$ in a REV

$\widehat{f}$	-	value f morpho-fluctuation in a Ω_f
H	-	width of the channel $[m]$
h	-	half-width of the channel $[m]$ or heat transfer coefficient $[W/m^2K]$
k	-	thermal conductivity $[W/(mK)]$
K_m	-	averaged turbulent eddy viscosity $[m^2/s]$
K_s	-	effective thermal conductivity of solid phase $[W/(mK)]$
K_T	-	turbulent eddy thermal conductivity $[W/(mK)]$
m	-	porosity [-]
$\langle m \rangle$	-	averaged porosity [-]

Nu	-	Nusselt number [-]
p	-	pressure $[Pa]$
Pe	-	Peclet number [-]
Pr	-	Prandtl number [-]
Re_{por}	-	Heat sink porous Reynolds number [-]
S_w	-	specific surface of a porous medium $\partial S_w/\Delta\Omega$ $[1/m]$
S_{wp}	-	$= S_\perp/\Delta\Omega$ $[1/m]$
$S_\perp$	-	cross flow projected area of obstacles $[m^2]$
T	-	temperature $[K]$
u, w	-	velocity in x,z-direction $[m/s]$
U	-	spacious average velocity in channel $[m/s]$
V	-	volume $[m^3]$
x	-	channel flow direction [-]
z	-	direction perpendicular to wall [-]

Subscripts

0	-	scale
$conv$	-	convection
$exchange$	-	heat exchange
f	-	fluid phase
i	-	component of variable vector
in	-	inlet
L	-	laminar
m	-	scale value
M	-	morphology
P	-	physical
por	-	porous
r	-	roughness
s	-	solid phase
$surf$	-	surface
T	-	turbulent
w	-	wall

Superscripts

$\sim$	-	value in fluid phase averaged over the REV
$-$	-	mean turbulent quantity
$\hat{}$	-	spacious fluctuation value in fluid phase
$*$	-	non-dimensional variable

Greek letters

$\widetilde{\alpha}_T$	-	averaged heat transfer coefficient over ∂S_w $[W/(m^2K)]$
$\Delta\Omega$	-	representative elementary volume (REV) $[m^3]$
$\Delta\Omega_f$	-	pore volume in a REV $[m_3]$
$\Delta\Omega_s$	-	solid phase volume in a REV $[m_3]$
ν	-	kinematic viscosity $[m^2/s]$
μ		dynamic viscosity $[kg/ms]$
ϱ	-	density $[kg/m^3]$

Introduction

The first attempt to develop a method for optimization of a heterogeneous, hierarchical scaled media was outlined by Travkin et al. (1994) and Travkin et al. (2000) using a volume averaged set of equations to obtain a 1D flow representation with 2D two-temperature simulation. The equations were closed using experimental correlations. Since then a number of papers have been published by these authors to document their progress, for example Gratton et al. (1996), Catton and Travkin (1997) and Travkin and Catton (1998). Their work covers a wide variety of transport phenomena ranging from fluid mechanics to crystal photonic band-gap problems (Travkin and Catton 2001). The theoretical development of transport phenomena in heterogeneous media with multiple scales has now been brought to the level where a specific application can be chosen for demonstration. The application chosen is enhancement of heat transfer dissipation from a given area of a flat surface while minimizing the frictional resistance (a problem of importance to all designers of heat exchangers). This problem has been under investigation for more than 3 decades and in spite of its longevity and importance as a problem, it has not been satisfactorily treated.

A majority of past investigations focused on the solution to a specific optimization task with a very limited number of spatial parameters to be varied, and usually a fixed geometric configuration, and tuned in their search for a maximum level of heat exchange (see, for example, Bejan and Morega (1993) and Ledezma et al. (1996)). This approach is a "single-scale" homogeneous approach yielding an optimum for a certain morphology and flow intensity without giving an explanation for why it was achieved. Without an explanation, there is no guidance on how to change the design to improve its performance. For each new morphology the experiment, whether real or numerical, needs to be performed again. In the heat exchanger industry there are countless research studies devoted to this problem.

Circuit density and power dissipation of integrated circuit chips are increasing, and more and more electrical devices are requiring some form of thermal management to adequately cool the chips. One of the more commonly used methods of improving thermal performance is to use heat sinks. In this work we outline how earlier work (Travkin and Catton (1998, 2001) and Gratton et al. (1996)) can be applied to optimize a semiconductor heat sink design. Optimal control variables for heat sink design are generated from VAT equations and controled by VAT equations. The variable design parameters are constrained between upper and lower bounds due to physical limitations. Computer aided numerical simulation can not yet replace the experimental work , but with the aid of computer calculations heat sink design can be focused on achieving recommended optimum properties. Model calculations can be used to examine the sensitivity of porous media performance to key performance parameters. This minimizes developmental costs and reduces the time required for product commercialization.

The present treatment of the heat sink optimization process can be applied to any specific hierarchical heterostructure with the aim to optimize its performance. What has been done is a demonstration of the only heterogeneous tool combining the model mathematical and morphological descriptions in one problem statement.

VAT Equations in the Form of Control Equations

The averaged laminar momentum equation (Travkin and Catton, 2001),

$$\begin{aligned} & \nu\frac{\partial}{\partial z}\left(\langle m(z)\rangle \frac{\partial \widetilde{U}(z)}{\partial z}\right) + U_{MConv} \\ & + U_{MFriction} - U_{MDrag} \\ = & \frac{1}{\rho_f}\frac{\partial\left(\langle m(z)\rangle \widetilde{p}\right)}{\partial x}, \end{aligned} \qquad (1)$$

is "controlled" by the three morphological terms that are defined as the "morpho-convective" fluctuation field distribution based term

$$U_{MConv}\left(\widehat{u},\ \widehat{w},\partial S_w,\Delta\Omega_f,\Delta\Omega_s\right)=\frac{\partial}{\partial z}\left(\left\langle -\widehat{u}\ \widehat{w}\right\rangle_f\right), \tag{2}$$

the interface surface skin friction term

$$U_{MFriction}\left(U,\partial S_w,\nu\right)=\frac{\nu}{\Delta\Omega}\int_{\partial S_w}\frac{\partial U}{\partial x_i}\cdot\overrightarrow{ds}, \tag{3}$$

and the solid phase drag resistance term

$$U_{MDrag}\left(p_j,\partial S_w\right)=\frac{1}{\varrho_f\Delta\Omega}\int_{\partial S_w}p\ \overrightarrow{ds}, \tag{4}$$

where the second left hand side term $\partial\left(\left\langle -\widehat{u}\ \widehat{w}\right\rangle_f\right)/\partial z$ presents cross-fluctuations effect. The presence of the vertical velocities - w and $\widetilde{w}$, or $\widehat{w}=w-\widetilde{w}$, seen in the first term, do not exist at the macrolevel because z direction momentum transport is only present close to obstacles. In traditional (homogeneous) one-scale shape optimization approaches these three terms are not present (see, for example, Ledezma et al. 1996) and, as a result, optimization methods are restricted in their value.

The laminar fluid energy equation is

$$\begin{aligned}&c_{pf}\varrho_f\left\langle m\right\rangle\widetilde{U}\frac{\partial\widetilde{T}_f}{\partial x}\\=\;&k\frac{\partial}{\partial x}\left[\frac{\partial\left\langle m\right\rangle\widetilde{T}_f}{\partial x}\right]+k\frac{\partial}{\partial z}\left[\frac{\partial\left\langle m\right\rangle\widetilde{T}_f}{\partial z}\right]\\&+T_{fMConvX}+T_{fMConvZ}+T_{fMSurfX}\\&+T_{fMSurfZ}+T_{fMExchange}.\end{aligned} \tag{5}$$

with the five additional control terms being

$$\begin{aligned}&T_{fMConvX}\left(\widehat{T}_f,\ \widehat{u},\Delta\Omega_f,\Delta\Omega_s\right)\\=\;&c_{pf}\varrho_f\frac{\partial}{\partial x}\left(\left\langle m\right\rangle\left\{-\widehat{T}_f\ \widehat{u}\right\}_f\right),\end{aligned} \tag{6}$$

$$\begin{aligned}&T_{fMConvZ}\left(\widehat{T}_f,\ \widehat{w},\Delta\Omega_f,\Delta\Omega_s\right)\\=\;&c_{pf}\varrho_f\frac{\partial}{\partial z}\left(\left\langle m\right\rangle\left\{-\widehat{T}_f\ \widehat{w}\right\}_f\right),\end{aligned} \tag{7}$$

$$T_{fMSurfX}\left(k,T_f,\ \partial S_w\right)=k\frac{\partial}{\partial x}\left[\frac{1}{\Delta\Omega}\int_{\partial S_w}T_f\ \overrightarrow{ds}\right], \tag{8}$$

$$T_{fMSurfZ}\left(k,T_f,\ \partial S_w\right)=k\frac{\partial}{\partial z}\left[\frac{1}{\Delta\Omega}\int_{\partial S_w}T_f\ \overrightarrow{ds}\right], \tag{9}$$

$$T_{fMExchange}\left(k,T_f,\ \partial S_w\right)=\frac{k}{\Delta\Omega}\int_{\partial S_w}\frac{\partial T_f}{\partial x_i}\cdot\overrightarrow{ds}\ . \tag{10}$$

Finally, the solid phase energy equation is

$$\begin{aligned}&k_s\frac{\partial}{\partial x}\left(\frac{\partial\left\langle s\right\rangle\left\{T_s\right\}_s}{\partial x}\right)+k_s\frac{\partial}{\partial z}\left(\frac{\partial\left\langle s\right\rangle\left\{T_s\right\}_s}{\partial z}\right)\\&+T_{sMSurfX}+T_{sMSurfZ}+T_{sMExchange}\\=\;&0,\end{aligned} \tag{11}$$

with the three control terms defined as follows:

$$T_{sMSurfX}\left(k_s,T_s,\ \partial S_w\right)=k_s\frac{\partial}{\partial x}\left[\frac{1}{\Delta\Omega}\int_{\partial S_w}T_s\ \overrightarrow{ds}_1\right], \tag{12}$$

$$T_{sMSurfZ}\left(k_s,T_s,\ \partial S_w\right)=k_s\frac{\partial}{\partial z}\left[\frac{1}{\Delta\Omega}\int_{\partial S_w}T_s\ \overrightarrow{ds}_1\right], \tag{13}$$

$$T_{sMExchange}\left(k_s, T_s,\ \partial S_w\right) = \frac{k_s}{\Delta\Omega}\int_{\partial S_w}\frac{\partial T_s}{\partial x_i}\cdot\vec{ds_1}\ . \tag{14}$$

In the turbulent regime, the momentum, fluid energy and solid energy equations are similar to what are shown but with an increased number of control terms and more complexity. They are not reproduced here and can be found in Gratton et al. (1996) and Travkin and Catton (2001). Some discussion about how they will be dealt with is found in the final section of this paper.

Description of Heat Transfer Enhancing Elements with Regular Morphology

The control equations are made general by non-dimensionalization with the following scaling,

$$\begin{aligned}
S_w &= S_w^* S_{wm},\ z = z_m z^*,\ z_m = \frac{4m_0}{S_{wm}},\ \widetilde{\overline{U}} = u_m u^*,\\
T_m &= \frac{z_m Q_0}{K_{Tm}},\ \nu = z_m u_m \nu^*, \langle m(z)\rangle = m_0\langle m^*\rangle,\\
c_d &= C_d^* c_{dm},\ c_{dm} = \frac{2u_m^2}{u_0^2}, \frac{1}{\varrho_f}\frac{d\langle p\rangle_f}{dx} = -\frac{u_m^2}{z_m},\\
k_{fm} &= z_m u_m c_{pf}\varrho_f,\ k_f = k_f^* z_m u_m c_{pf}\varrho_f,\\
k_s &= k_s^* z_m u_m c_p \varrho_s.
\end{aligned} \tag{15}$$

The parameters resulting for laminar flow through a morphology that is constant normal to the flow direction are given in the table 1 with their possible ranges. These parameters are at the discretion of the designer of a heat transfer device and can be used for optimization.

The first two parameters represent the influence of medium resistance and morphology features like porosity m_0, the next two paramenters are the porous medium Peclet number $Pe_m = z_m u_m/\alpha_f = \left(\frac{4m_0}{S_{wm}}\left(-\frac{z_m}{\rho_f}\frac{d\langle p\rangle_f}{dx}\right)^{1/2}\right)/\alpha_f$, and heat exchange rate between the phases $\alpha_L^* S_w^* = (\alpha_L S_w)/(\alpha_{Lm} S_{wm}) = (\alpha_L S_w)/\left(u_m c_{pf}\rho_f S_{wm}\right)$, and the last two parameters are L_{P7N}, the solid phase temperature governing equation heat exchange parameter, and L_{B8}, a II kind simplified boundary condition parameter (see Travkin and Catton (1995) for details).

var.	definition	min	max
L_{3N}	$= Re_{mf} c_{dm} C_d^* S_w^*$	10^{-5}	5×10^7
L_{M4N}	$= Re_{mf}(1/m_0)$	10^{-3}	10^5
L_{P5}	$= \frac{1}{Pe_m}$	2.1	2×10^7
L_{P6}	$= \alpha_L^* S_w^*$	1.0	10^8
L_{P7N}	$= \frac{Pe_m}{A_k(L_{M4}-1)}\alpha_L^* S_w^*$	0	10^{20}
L_{B8}	$= \frac{A_k}{Pe_m} = A_k L_{p5}$	10^{-3}	10^{12}

Table 1: Optimization variables and their ranges for laminar flow in porous media

There are six nondimensional control parameters and functions, denoted Medium Specific Control Functions (MSCF), that control the heat and momentum transport in the selected porous medium and that can be modified to optimize the performance. The two terms with the broadest range also have the greatest influence on the outcome.

If the morphology functions denoting porosity, $\langle m(z)\rangle$, and specific surface area, $S_w(z)$,are coordinate specific, then the equations and and parameters sets are different yielding eight control parameters instead of six. A similar exersize for turbulent flow with $\langle m\rangle = const,\ S_w = const$ will yield the eight optimization parameters shown in table 2.

Preliminary Optimization Methods Using the Variation of Selected Parameters and Design of Experiment Statistical Approach

Some previous simulation results using VAT based transport and closure models for flow in a channel with rib roughened walls; spherical beads, round tube banks and square tube banks yielded optimal configurations, see Travkin et

var.	definition	min	max
L_{P2}	$= K_w = \frac{\nu}{z_m u_m} = \frac{1}{Re_{mf}}$	10^{-3}	10^{3}
L_3	$= c_{dm} C_d^* S_w^*$	10^{-2}	5×10^{6}
L_{M4}	$= A_4 = 1/m_0$	1.0	10^{3}
L_{P5}	$= \alpha_T^* \left(u^*, m^*\right) S_w^* \left(z^*\right)$	1.0	10^{8}
L_{P6N}	$= \frac{Pe_m}{A_k (L_{M4}-1)} \alpha_T^* S_w^*$	0	10^{20}
L_{P7}	$= \ \sigma_b = \frac{\widetilde{K}_m}{K_b}$	0.1	10
L_{B8}	$= \frac{A_k a_f}{z_m u_m} = \frac{A_k}{Pe_m} = A_k L_{p5}$	10^{-3}	10^{12}
L_{B9}	$= \frac{1}{Pe_{TB}} = K_{TB}^* = \frac{a_{TB}}{z_m u_m}$	10^{-1}	10^{3}

Table 2: Optimization variables and their ranges for turbulent flow in porous media

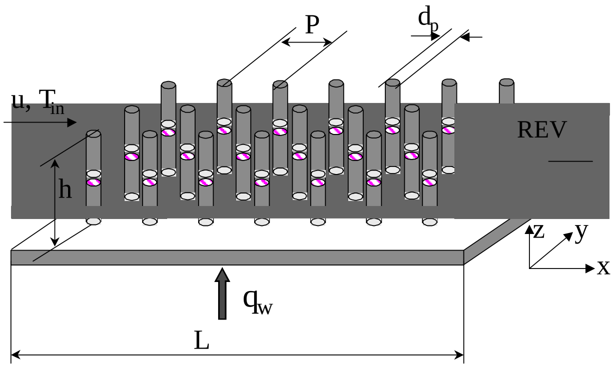

Figure 1: Pin fin heat sink

al. (1998, 2001). Through the application of the VAT closure model to some general morphology models, orifice and plane slit, in limiting cases, it was demonstrated that both the transport model and the closure scheme are reasonable. The numerical results show that the VAT based model is applicable to the flow in channels with rough walls or in channels filled with a regular porous matrix, and demonstrates how the simplest morphological properties of a porous layer such as porosity function and specific surface along with closure models naturally affects the transport features and can be helpful for preliminary investigation of the optimized morphologies. The morphology models used in the numerical simulations are shown in Fig. 1.

Varing the physical parameters of the heat sink shown in Fig. 1 changes the values of the variables shown in table 1 and table 2. Using volume averaging theory, heat sink porosity is calculated from

$$\langle m \rangle = \frac{P^2 - \frac{1}{4}\pi d_p^2}{P^2}. \tag{16}$$

The heat sink specific surface is

$$S_w = \frac{\pi d_{fin}}{P^2} \tag{17}$$

While the characteristic length for the heat sink is defined as

$$d_{por} = \frac{4\langle m \rangle}{S_w}, \tag{18}$$

which is the volumetric equivelent length. Closure for momentum and heat transport in pin fin heat sinks was developed from the analysis of Zukauskas (1987). Heat sink drag coefficient and heat transfer coefficient are determined from

$$C_d = \left(\frac{2\Delta p}{\varrho_f \tilde{\bar{u}}^2 nP}\right)\left(\frac{\langle m \rangle}{S_w}\right) \tag{19}$$

and

$$h = \frac{0.27 Re_{por}^{0.63} Pr^{0.36} k_f}{d_{por}} \tag{20}$$

respectively.

To optimize a heat sink, the first step is to evaluate its performace. Generally, evaluation of heat sink is simple, that is increasing heat transfer coefficient and decreasing momentum resistance. But as long as additional heat exchanging elements such as fins and ribs are used to maximize heat transfer rate within specific volume of heat sink, the problem of heat sink evaluation and optimization becomes a two scale heterogeneous problem. So the evaluation method should account for heat transfer performance, pressure drop penalty, and heterogeneous medium properties as well as fluid properties. Dividing heat sink heat transfer rate per unit volume by heat sink pumping power cost per unit volume yields the form

$$E_{eff} = \left(\frac{Nu_w}{f_f \, \mathrm{Re}_{por}^3} \right) \left(32 \frac{S_{all}}{\Omega \left\langle m_{yz} \right\rangle} \frac{\left\langle m \right\rangle^3}{S_w^3} \right) \left(\frac{k_f \varrho_f^2}{\mu^3} \right), \quad (21)$$

which is a combination of Nusselt number, the friction factor the pore Reynolds number, heterogeneous medium properties and fluid properties. Nu_w is the Nusselt number across the channel, and is defined as

$$Nu_w = \frac{q_w d_{por}}{\left(T_{w\max} - T_{in} \right) k_f}. \quad (22)$$

S_{all} is the total heat transfer area of heat sink including bottom surface area and internal surface area. Fig. 2 shows the dependence of the heat sink effectivness number, E_{eff}, on the porosity for different morphologies at different Reynolds numbers. The parameters shown in Fig. 1 are chosen as $d_p = 3.175mm, h = 38mm, L = 100mm$ and base thickness $= 6mm$. Changing fin pitch yields different porosity of the heat sink. When the porosity increases, fins play less role on pressure loss. This dramatically decreases pressure loss and increases heat sink effectiveness. Fig. 3 shows the variation of heat sink Nusselt number for different porosity and different Reynolds number. It is obvious Nusselt number is high when Reynolds number is high. For high Reynolds number, heat transfer performace will be best at certain porosity.

There are many other considerations when a heat exchange device is optimized. For example, electronic devices often have serious space limitations and it is volume one might want to minimize. Another choice might be the amount of material used in a heat sink. To properly

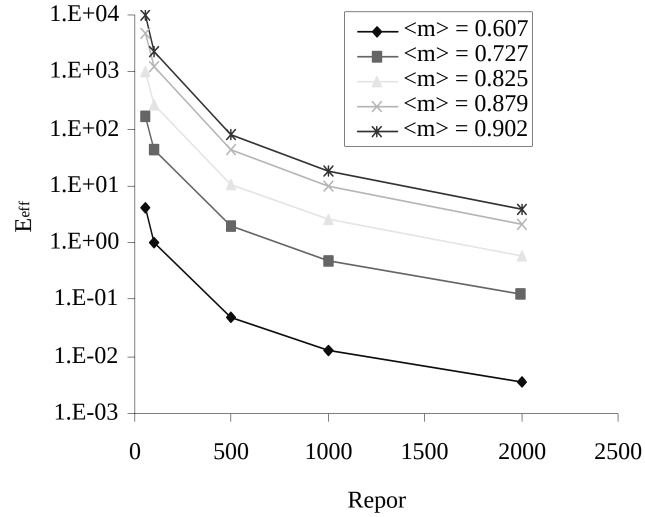

Figure 2: Heat sink heterogeneous effectiveness

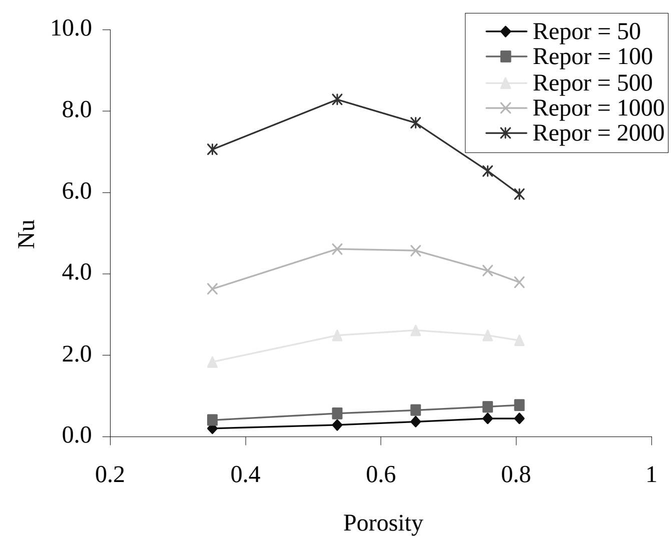

Figure 3: Heat sink wall Nusselt number when changing heat sink pin fin pitch

optimize such devices, one must weigh all the paramters at the outset. Once it is decided what it is that is to be optimized, the process is the same.

When the problem becomes multi-dimensional, 6D or 8D, according to Atwood (1969), Diamond (1981), Box and Draper (1987) and Montgomery (1991), it is convenient to use statistical design of experiment (DOE) methodology. An optimal response surface was found in two steps. First, numerical simulation was carried out based on statistical selection of the parameter values. Second a statistical analysis of the results was used to develop a response surface. This procedure was implemented using a commercial computer code based on DOE.

When the optimization variable is choosen, in our case E_{eff},the variables are systematically defined, see the table of parameters developed above. Next, the numerical experiment design type is selected,e.g. a classical two level, mixed level,or nested level. The design type used in this work is the classical two level design. The classical two level designs are based on standard orthogonal arrays that contain two levels for each experimental variable. It enables estimation of the effects of some or all terms in a second order model of the general form

$$\begin{aligned} E_{eff} &= a_0 + a_1 X_1 + a_2 X_2 + ... + a_n X_n \\ &\quad + a_{11} X_1^2 + a_{12} X_1 X_2 + ... \\ &\quad + a_{n,n-1} X_n X_{n-1} + a_{nn} X_n^2 \qquad (23) \end{aligned}$$

The independent variables $X_{1,}X_{2,}...X_n$ are the design variables $L_{3N}...L_{B8}$ or $L_{P2}...L_{B9}$. Based on the design type and design variables, experiment design options will be created. Each option is a set of input parameters for numerical simulation. Description of what was done to obtain the "experimental results" from the VAT based laminar or turbulent transport equations for flow in a specific porous media is decribed elsewhere (see Travkin and Catton (1995), Travkin et al. (1998)).

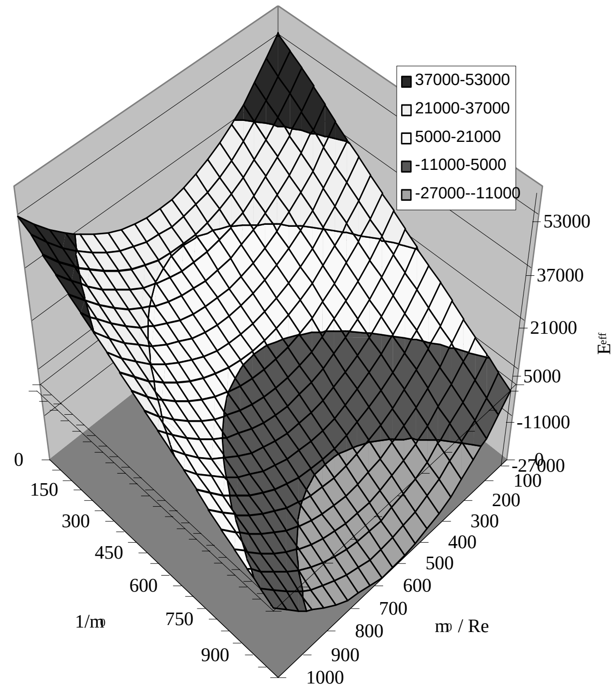

Figure 4: Heat sink optimization response surface

After numerical simulation, the numerical results are rigorously analyzed using statistical analysis tools and graphics tools. These tools include nonlinear response/error analysis, experimental error analysis, regression analysis, residuals analysis, two dimensional graphing, three dimensional response surface graphing, and multi response optimization. One of the response surfaces of our study is shown in Fig. 4. The three dimensional figure shows E_{eff} as a function of two variables L_{P2} and L_{M4} (these were chosen for simplicity from the eight independent varables analysed) when the other variables are fixed. Although limited by the range of the variables, the optimum point is shown on the figure.and the trend of the response surface is clearly shown in Fig. 4.

Two-Scale Optimization Study

Closure of the turbulent regime VAT equations for a porous flat channel also requires closure of additional terms in the governing equations. This is done (as for laminar regime) using Direct Numerical Modeling (DNM). The four terms arising in the momentum equation are

$$\frac{\partial}{\partial z}\left(\left\langle \widehat{K}_m \frac{\partial \widehat{\widehat{u}}}{\partial z}\right\rangle_f\right), \tag{24}$$

$$\frac{\partial}{\partial z}\left(\left\langle -\widehat{\widehat{u}}\,\widehat{\widehat{w}}\right\rangle_f\right),$$

$$\frac{1}{\Delta\Omega}\int_{\partial S_w}(K_m+\nu)\frac{\partial \overline{U}}{\partial x_i}\cdot \vec{ds},$$

$$\frac{1}{\varrho_f\Delta\Omega}\int_{\partial S_w}\overline{p}\,\vec{ds},$$

the seven terms in the fluid temperature equation

$$\frac{\partial}{\partial x}\left(\left\langle \widehat{K}_T \frac{\partial \widehat{\widehat{T}}_f}{\partial x}\right\rangle_f\right), \tag{25}$$

$$\frac{\partial}{\partial z}\left(\left\langle \widehat{K}_T \frac{\partial \widehat{\widehat{T}}_f}{\partial z}\right\rangle_f\right),$$

$$c_{pf}\varrho_f\frac{\partial}{\partial x}\left(\langle m\rangle\left\{-\widehat{\widehat{T}}_f\,\widehat{\widehat{u}}\right\}_f\right),$$

$$c_{pf}\varrho_f\frac{\partial}{\partial z}\left(\langle m\rangle\left\{-\widehat{\widehat{T}}_f\,\widehat{\widehat{w}}\right\}_f\right),$$

$$\frac{\partial}{\partial x}\left[\frac{\left(\widetilde{K}_T+k\right)}{\Delta\Omega}\int_{\partial S_w}\overline{T}_f\,\vec{ds}\right],$$

$$\frac{\partial}{\partial z}\left[\frac{\left(\widetilde{K}_T+k\right)}{\Delta\Omega}\int_{\partial S_w}\overline{T}_f\,\vec{ds}\right],$$

$$\frac{1}{\Delta\Omega}\int_{\partial S_w}(K_T+k)\frac{\partial \overline{T}_f}{\partial x_i}\cdot \vec{ds}$$

and five terms in the solid phase temperature equation

$$\frac{\partial}{\partial x}\left[\left\langle \widehat{K}_s \frac{\partial \widehat{T}_s}{\partial x}\right\rangle_s\right], \tag{26}$$

$$\frac{\partial}{\partial z}\left[\left\langle \widehat{K}_s \frac{\partial \widehat{T}_s}{\partial z}\right\rangle_s\right],$$

$$\frac{\partial}{\partial x}\left[\frac{\{K_s\}_s}{\Delta\Omega}\int_{\partial S_{w12}}T_s\,\vec{ds}_1\right],$$

$$\frac{\partial}{\partial z}\left[\frac{\{K_s\}_s}{\Delta\Omega}\int_{\partial S_{w12}}T_s\,\vec{ds}_1\right],$$

$$\frac{1}{\Delta\Omega}\int_{\partial S_{w12}}K_{sT}\frac{\partial T_s}{\partial x_i}\cdot \vec{ds}_1$$

The mathematical implementation needed to obtain closure of the momentum resistance terms needed for optimization of the morphology of heat sink; is dictated by the geometry of the fins. For example, if on the side straight vertical interface surfaces the surface element $\vec{ds}_1=\vec{n}\,ds$, $\vec{n}=(-\mathbf{i},0,0)|_{\partial S_{wL}}$ and $\vec{n}=(\mathbf{i},0,0)|_{\partial S_{wR}}$, then the surface integral over the ∂S_w in each of the intermediate REVs not including portions of the free volume above the fins and those at the bottom of the solid phase of the channel will be, for the x-coordinate frictional resistance component,

$$\begin{aligned}
&\frac{1}{\Delta\Omega}\int_{\partial S_w}(K_m+\nu)\,\mathbf{n}_i\cdot\frac{\partial U_j}{\partial x_i}ds \\
=\;&\frac{1}{\Delta\Omega}\int_{\partial S_w}(K_m+\nu)\left(n_x\frac{\partial U}{\partial x}+n_y\frac{\partial U}{\partial y}+n_z\frac{\partial U}{\partial z}\right)ds\mathbf{i} \\
=\;&\left(\frac{-1}{\Delta\Omega}\int_{z_{kL}}^{z_{(k+1)L}}\left[(K_m+\nu)\frac{\partial U}{\partial x}\right]_{\partial S_{wL}}dz\right. \\
&\left.+\frac{1}{\Delta\Omega}\int_{z_{kR}}^{z_{(k+1)R}}\left[(K_m+\nu)\frac{\partial U}{\partial x}\right]_{\partial S_{wR}}dz\right)\mathbf{i}.
\end{aligned} \tag{27}$$

Our analysis of many existing morphological solutions has led us to conclude that the scaled hierarchical VAT description gives us the ability to find an optimum morphology that cannot be improved when the selection of fluids and solid phase materials has been made, and the pressure drop through the media is specified. Given these initial conditions (restrictions), it is possible to find a morphology that cannot be improved based on two scale heat transport meaning that there is no other solid phase configuration that can be more efficient than the one that has been found.

Summary

In this brief paper we have illustrated a method of hierarchical optimization of two- and three scale heat transport in a heterogeneous meda. It is shown how traditional governing equations developed using rigorous VAT methods can be used to optimize surface transport processes in support of heat transport technology.

The difficulty in treating a multiparameter (more than 3) problem, even linear, known to be very difficult to overcome using a parameter sorting process. The combination of VAT based equations and the theory of statistical design was used to effectively begin treating 6D or 8D optimization volumes.

We have shown how a two scale heterogeneous heat transfer optimization problem can be solved using exact procedures for closure of additional differential and integral VAT terms. This method is shown to be as simple as calculating the appropriate integrals over the morphologies with coordinate surfaces of interfaces pertinent to a morphology of interest. For more complex or even unknown morphologies as initial spacial morphologies, the mathematical methods were outlined in detail. These three tasks were carried out, albeit for some elementary morphologies, for the first time. This project is still on going. The results presented here are preliminary results.

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