solutions of the linear quadratic regulator problem, including conditions for the convergence of modal approximation schemes. However, for more general optimal control problems involving PDEs, the main approach has been to use some method for constructing a particular finite-dimensional approximating optimal control problem and then to solve this problem by some method or other (Teo and Wu [213]).

It seems that no attention has been given to the optimal control systems governed by the partial integrodifferential equations like volume averaging theory equations for HE design.

B. New Kinds of Heat Exchanger Mathematical Models

Our earlier work has shown that flow resistance and heat transfer in HEs and CHEs can be treated as highly porous structures and that their behavior can be properly predicted by averaging the transport equations over a representative elementary volume (REV) in the region neighboring the surface. The averaging of processes in regular and randomly organized heterogeneous media and in HE can be performed in different ways. Travkin and Catton [21, 28] discussed alternate forms for the mass, momentum, and heat transport equations recently presented by various researchers. The alternate forms of the transport equations are often quite different. The differences among the transport equation forms advocated by the numerous authors demonstrate the fact that research on the basic form of the governing equations of transport processes in heterogeneous media is still an evolving field of study. Derivation of the equations of flow and heat transport for a highly porous medium during the filtration mode is based on the theory of averaging by certain REV of the transfer equation in the liquid phase and transfer equations in the solid phase of the heterogeneous medium (see, for example, Whitaker [42, 10] for laminar regime developments, and Shcherban *et al.* [15], Primak *et al.* [14], and Travkin and Catton [16, 21, 23] for turbulent filtration).

These models account for the medium morphology characteristics. Using second-order turbulent models, equation sets are obtained for turbulent filtration and two-temperature diffusion in nonisotropic porous media with interphase exchange and micro-roughness. The equations differ from those found in the literature. They were developed using an advanced averaging technique, a hierarchical modeling methodology, and fully turbulent models with Reynolds stresses and fluxes in every pore space.

Independent treatment of turbulent energy transport in the fluid phase and energy transport in the solid phase, connected through the specific surface (the solid–fluid interface in the REV), allows for more accurate modeling of the heat transfer mechanisms between rough surfaces or porous insert of HE and the fluid phases.

C. VAT-Based Compact Heat Exchanger Modeling

For a pin fin (PFHE), with cross-flow morphology, the governing equations can be written in the following form:

Momentum equation for the first fluid:

$$\frac{\partial}{\partial x}\left(\langle m_1\rangle(\tilde{K}_{m1}+\nu_1)\frac{\partial\tilde{\bar{U}}_1}{\partial x}\right)+\frac{\partial}{\partial x}\left(\left\langle \hat{K}_{m1}\frac{\partial\hat{\bar{u}}_1}{\partial x}\right\rangle_f\right)+\frac{\partial}{\partial x}(\langle -\hat{u}_1\hat{u}_1\rangle_f)$$
$$=\langle m_1\rangle\tilde{\bar{U}}_1\frac{\tilde{\bar{U}}_1}{\partial x}-\frac{1}{\Delta\Omega}\int_{\partial S_{w1}}(K_{m1}+\nu_1)\frac{\partial\bar{U}_1}{\partial x_i}\cdot\vec{d}s$$
$$+\frac{1}{\rho_{f1}\Delta\Omega}\int_{\partial S_{w1}}\bar{p}_1\vec{d}s+\frac{1}{\rho_{f1}}\frac{\partial}{\partial x}(\langle m_1\rangle\tilde{\bar{p}}_1). \qquad (432)$$

momentum equation for the second fluid:

$$\frac{\partial}{\partial z}\left(\langle m_2\rangle(\tilde{K}_{m1}+\nu_2)\frac{\partial\tilde{\bar{W}}_2}{\partial z}\right)+\frac{\partial}{\partial z}\left(\left\langle \hat{K}_{m2}\frac{\partial\hat{\bar{w}}_2}{\partial z}\right\rangle_f\right)+\frac{\partial}{\partial z}(\langle -\hat{w}_2\hat{w}_2\rangle_f)$$
$$=\langle m_2\rangle\tilde{\bar{W}}_2\frac{\partial\tilde{\bar{W}}_2}{\partial z}-\frac{1}{\Delta\Omega}\int_{\partial S_{w2}}(K_{m2}+\nu_2)\frac{\partial\bar{W}_2}{\partial x_i}\cdot\vec{d}s$$
$$+\frac{1}{\rho_{f2}\Delta\Omega}\int_{\partial S_{w2}}\bar{p}_2\vec{d}s+\frac{1}{\rho_{f2}}\frac{\partial}{\partial z}(\langle m_2\rangle\tilde{\bar{p}}_2). \qquad (433)$$

Energy equation for the first fluid:

$$c_{pf1}\rho_{f1}\langle m_1\rangle\tilde{\bar{U}}_1\frac{\partial\tilde{\bar{T}}_1}{\partial x}=\frac{\partial}{\partial x}\left[\langle m_1\rangle(\tilde{K}_{T1}+k_1)\frac{\partial\tilde{\bar{T}}_1}{\partial x}\right]$$
$$+\frac{\partial}{\partial z}\left[\langle m_1\rangle(\tilde{K}_{T1}+k_1)\frac{\partial\tilde{\bar{T}}_1}{\partial z}\right]$$
$$+\frac{\partial}{\partial x}\left(\left\langle\hat{K}_{T1}\frac{\partial\tilde{\bar{T}}_1}{\partial x}\right\rangle_f\right)+\frac{\partial}{\partial z}\left(\left\langle\hat{K}_{T1}\frac{\partial\tilde{\bar{T}}_1}{\partial z}\right\rangle_f\right)$$
$$+c_{pf1}\rho_{f1}\frac{\partial}{\partial x}(\langle m_1\rangle\{-\tilde{\bar{T}}_1\hat{u}_1\}_f)$$
$$+\frac{\partial}{\partial x}\left[\frac{(\tilde{K}_{T1}+k_1)}{\Delta\Omega}\int_{\partial S_{w1}}\hat{\bar{T}}_1\vec{d}s\right]$$
$$+\frac{\partial}{\partial z}\left[\frac{(\tilde{K}_{T1}+k_1)}{\Delta\Omega}\int_{\partial S_{w1}}\hat{\bar{T}}_1\vec{d}s\right]$$
$$+\frac{1}{\Delta\Omega}\int_{\partial S_{w1}}(K_{T1}+k_1)\frac{\partial\bar{T}_1}{\partial x_i}\cdot\vec{d}s. \qquad (434)$$

Energy equation for the solid phase:

$$\frac{\partial}{\partial x}\left(\langle s\rangle\{K_{sT}\}_s\frac{\partial\{T_s\}_s}{\partial x}\right)+\frac{\partial}{\partial x}\left(\left\langle \hat{K}_{sT}\frac{\partial \hat{T}_s}{\partial x}\right\rangle_s\right)+\frac{\partial}{\partial z}\left(\langle s\rangle\{K_{sT}\}_s\frac{\partial\{T_s\}_s}{\partial x}\right)$$
$$+\frac{\partial}{\partial z}\left(\left\langle \hat{K}_{sT}\frac{\partial\hat{T}_s}{\partial x}\right\rangle_s\right)+\frac{\partial}{\partial x}\left[\frac{\{K_{sT}\}_s}{\Delta\Omega}\int_{\partial S_{w12}}\hat{T}_s d\vec{s}_1\right]$$
$$+\frac{\partial}{\partial z}\left[\frac{\{K_{sT}\}_s}{\Delta\Omega}\int_{\partial S_{w12}}\hat{T}_s d\vec{s}_1\right]+\frac{1}{\Delta\Omega}\int_{\partial S_{w12}} K_{sT}\frac{\partial T_s}{\partial x_i}\cdot d\vec{s}_1=0. \qquad (435)$$

Energy equation for the second fluid:

$$c_{pf2}\rho_{f2}\langle m_2\rangle \tilde{\bar{W}}_2 \frac{\partial \tilde{\bar{T}}_2}{\partial z}=\frac{\partial}{\partial x}\left[\langle m_2\rangle(\tilde{K}_{T2}+k_2)\frac{\partial \tilde{\bar{T}}_2}{\partial x}\right]$$
$$+\frac{\partial}{\partial z}\left[\langle m_2\rangle(\tilde{K}_{T2}+k_2)\frac{\partial \tilde{\bar{T}}_2}{\partial z}\right]$$
$$+\frac{\partial}{\partial x}\left(\left\langle\hat{K}_{T2}\frac{\partial\tilde{\bar{T}}_2}{\partial x}\right\rangle_f\right)+\frac{\partial}{\partial z}\left(\left\langle\hat{K}_{T2}\frac{\partial\tilde{\bar{T}}_2}{\partial z}\right\rangle_f\right)$$
$$+c_{pf2}\rho_{f2}\frac{\partial}{\partial z}\left(\langle m_2\rangle\{-\tilde{\bar{T}}_2\hat{w}_2\}_f\right)$$
$$+\frac{\partial}{\partial x}\left[\frac{(\tilde{K}_{T2}+k_2)}{\Delta\Omega}\int_{\partial S_{w2}}\hat{\bar{T}}_2 d\vec{s}\right]$$
$$+\frac{\partial}{\partial z}\left[\frac{(\tilde{K}_{T2}+k_2)}{\Delta\Omega}\int_{\partial S_{w2}}\hat{\bar{T}}_2 d\vec{s}\right]$$
$$+\frac{1}{\Delta\Omega}\int_{\partial S_{w2}}(K_{T2}+k_2)\frac{\partial \bar{T}_2}{\partial x_i}\cdot d\vec{s}. \qquad (436)$$

The volumes for averaging in equations are $\Delta\Omega$, $\Delta\Omega_{f1}$, $\Delta\Omega_{f2}$, $\Delta\Omega_s$.

A majority of the additional terms in these equations can be treated using closure procedures developed in previous work (see, for example, Travkin and Catton [16, 19]), for selected fin geometries and solid matrices of a HE. Our generic interest, however, is in the theoretical applications of the VAT governing equations and possible advantages gained by introduction of irregular or random morphology into heat exchange volumes and surfaces.

Cocurrent parallel flow matrix type CHE morphology can be described using the next VAT-based set of governing equation.

Momentum equation for the first fluid:

$$\langle m_1\rangle \bar{\bar{U}}_1 \frac{\partial \bar{\bar{U}}_1}{\partial x} - \frac{1}{\Delta\Omega}\int_{\partial S_{w1}}(K_{m1}+\nu_1)\frac{\partial \bar{U}_1}{\partial x_i}\cdot d\vec{s} + \frac{1}{\rho_{f1}\Delta\Omega}\int_{\partial S_{w1}}\bar{p}_1 d\vec{s}$$
$$= -\frac{1}{\rho_{f1}}\frac{\partial}{\partial x}(\langle m_1\rangle\tilde{\bar{p}}_1)+\frac{\partial}{\partial x}\left(\langle m_1\rangle(\tilde{K}_{m1}+\nu_1)\frac{\partial\bar{\bar{U}}_1}{\partial z}\right)$$
$$+\frac{\partial}{\partial x}\left(\left\langle\hat{K}_{m1}\frac{\partial\hat{\bar{u}}_1}{\partial x}\right\rangle_f\right)+\frac{\partial}{\partial x}(\langle-\hat{\bar{u}}_1\hat{\bar{u}}_1\rangle_f). \qquad (438)$$

Momentum equation for the second fluid:

$$\langle m_2\rangle \bar{\bar{U}}_2 \frac{\partial \bar{\bar{U}}_2}{\partial x} - \frac{1}{\Delta\Omega}\int_{\partial S_{w2}}(K_{m2}+\nu_2)\frac{\partial \bar{U}_2}{\partial x_i}\cdot d\vec{s} + \frac{1}{\rho_{f2}\Delta\Omega}\int_{\partial S_{w2}}\bar{p}_2 d\vec{s}$$
$$= -\frac{1}{\rho_{f2}}\frac{\partial}{\partial x}(\langle m_2\rangle\tilde{\bar{p}}_2)+\frac{\partial}{\partial x}\left(\langle m_2\rangle(\tilde{K}_{m2}+\nu_2)\frac{\partial\bar{\bar{U}}_2}{\partial x}\right)$$
$$+\frac{\partial}{\partial x}\left(\left\langle\hat{K}_{m2}\frac{\partial\hat{\bar{u}}_2}{\partial x}\right\rangle_f\right)+\frac{\partial}{\partial x}(\langle-\hat{\bar{u}}_2\hat{\bar{u}}_2\rangle_f). \qquad (437)$$

The corresponding energy equations are like those given earlier. A simple example typifies the general morphology of cocurrent and countercurrent CHEs when widths of the channels are different and the heat transfer enhancing devices are to be determined by shape optimization. For this purpose, consider two conjugate flat channels of different heights that are both filled with unknown (or assigned) heat transfer elements or porous media. A set of governing equations for each of the channels were developed by Travkin and Catton ([16, 20]).

A model of the momentum equation for a horizontally homogeneous stream under steady conditions has the form

$$\frac{\partial}{\partial z}\left(\langle m\rangle(\tilde{K}_{mj}+\nu_j)\frac{\partial\bar{\bar{U}}_j}{\partial z}\right)+\frac{\partial}{\partial z}\left(\left\langle\hat{K}_{mj}\frac{\partial\hat{\bar{u}}_j}{\partial z}\right\rangle_f\right)+\frac{\partial}{\partial z}(\langle-\hat{\bar{u}}_j\hat{\bar{w}}_j\rangle_f)$$
$$+\frac{1}{\Delta\Omega}\int_{\partial S_{wrj}}K_{mj}\frac{\partial\bar{U}_j}{\partial x_i}\cdot d\vec{S}+\frac{1}{\Delta\Omega}\int_{\partial S_{wlj}}\nu_j\frac{\partial\bar{U}_j}{\partial x_i}\cdot d\vec{S}$$
$$-\frac{1}{\rho_{fj}\Delta\Omega}\int_{\partial S_{wj}}\bar{p}_j d\vec{S}=\frac{1}{\rho_{fj}}\frac{\partial\langle\bar{p}\rangle_{fj}}{\partial x}. \qquad (439)$$

This equation can be further simplified for turbulent flow in a layer with a porous filling or insert that has regular morphology,

$$\frac{\partial}{\partial z}\left(\langle m(z)\rangle(\tilde{K}_{mj}+\nu_j)\frac{\partial\tilde{\bar{U}}_j(z)}{\partial z}\right)+U_{jMT}(\bar{U}_j,\partial S_w,K_{mj})+U_{jML}(\bar{U}_j,\partial S_w,\nu_j)$$

$$+U_{jMform}(\bar{p}_j,\partial S_w)=\frac{1}{\rho_{fj}}\frac{\partial(\langle m(z)\rangle\tilde{\bar{p}}_j)}{\partial x}, \qquad (440)$$

where the three morphology-based terms are defined by

$$U_{jMT}(\bar{U}_j,\partial S_w,K_{mj})=\frac{1}{\Delta\Omega}\int_{\partial S_{wTj}}K_{mj}\frac{\partial\bar{U}_j}{\partial x_i}\cdot d\vec{S} \qquad (441)$$

$$U_{jML}(\bar{U}_j,\partial S_w,\nu_j)=\frac{1}{\Delta\Omega}\int_{\partial S_{wLj}}\nu_j\frac{\partial\bar{U}_j}{\partial x_i}\cdot d\vec{S} \qquad (442)$$

$$U_{jMform}(\bar{p}_j,\partial S_w)=-\frac{1}{\rho_{fj}\Delta\Omega}\int_{\partial S_{wj}}\bar{p}_j d\vec{S}. \qquad (443)$$

It is obvious that the result is "controlled" by three morphology terms.

The equation for the mean turbulent fluctuation energy $b(z)$ is written in the following simple form, which includes the effect of obstacles in the flow and temperature stratification across the layer, the z direction:

$$\tilde{K}_{mj}(z)\left(\frac{\partial\tilde{\bar{U}}_j}{\partial z}\right)^2+\frac{d}{dz}\left(\left(\frac{\tilde{K}_{mj}}{\sigma_b}+\nu_j\right)\frac{db_j(z)}{dz}\right)+\frac{f_{1j}(c_d)S_{wj}(z)}{\langle m\rangle}\tilde{\bar{U}}_j^3$$

$$-\frac{g}{T_a\sigma_T}\left[\tilde{K}_{mj}\frac{\partial\tilde{\bar{T}}_j}{\partial z}\right]+2\nu\left(\frac{db_j^{1/2}(z)}{dz}\right)^2=C_1\frac{b_j^2(z)}{\tilde{K}_{mj}}. \qquad (444)$$

Here, $f_1(c_d)$ is approximately the friction factor for constant and nearly constant morphology functions, and the mean eddy viscosity is given by

$$\tilde{K}_{mj}(z)=C_1^{1/4}l(z)b_j^{1/2}(z), \qquad (445)$$

where $l(z)$ is the turbulent scale function defined by the assumed porous medium structure.

Similarly, the equation of turbulent heat transfer in the homogeneous porous medium fluid phase is

$$c_{pfj}\rho_{fj}\langle m_j\rangle\tilde{\bar{U}}_j(z)\frac{\partial\tilde{\bar{T}}_j(x,z)}{\partial x}=\frac{\partial}{\partial z}\left(\langle m_j\rangle(\tilde{K}_{Tj}+k_{fj})\frac{\partial\tilde{\bar{T}}_j(x,z)}{\partial z}\right)$$

$$+T_{jMTqin}(\bar{T}_j,\partial S_w,K_{Tj})+T_{jMLqin}(\bar{T}_j,\partial S_w,k_j)$$

$$+\frac{1}{\Delta\Omega}\int_{\partial S_{wLj}}k_f\frac{\partial\bar{T}_j}{\partial x_i}\cdot d\vec{S}, \qquad (446)$$

with two morphology terms that "control" the solution being

$$T_{jMTqin}(\bar{T}_j,\partial S_w,K_{Tj})=\frac{1}{\Delta\Omega}\int_{\partial S_{wTj}}K_{Tj}\frac{\partial\bar{T}_j}{\partial x_i}\cdot d\vec{S} \qquad (447)$$

$$T_{jMLqin}(\bar{T}_j,\partial S_w,k_j)=\frac{1}{\Delta\Omega}\int_{\partial S_{wLj}}k_j\frac{\partial\bar{T}_j}{\partial x_i}\cdot d\vec{S}. \qquad (448)$$

In the solid phase of CHE, the energy equation is

$$\frac{\partial}{\partial z}\left((1-\langle m\rangle)\tilde{K}_{sT}(z)\frac{\partial\tilde{T}_s(x,z)}{\partial z}\right)+T_{sMqin}(T_s,\partial S_w,K_{sT})=0, \qquad (449)$$

with the one "control" term

$$T_{sMqin}(T_s,\partial S_w,K_{sT})=\frac{1}{\Delta\Omega}\int_{\partial S_{w12}}K_{sT}\frac{\partial T_s}{\partial x_i}\cdot d\vec{S},$$

where

$$d\vec{S}_1=-d\vec{S}.$$

If we apply the closure procedures described earlier, the equation of motion becomes

$$\frac{\partial}{\partial z}\left(\langle m(z)\rangle\tilde{K}_{mj}(\tilde{\bar{U}},b,l)\frac{\partial\tilde{\bar{U}}_j(z)}{\partial z}\right)$$

$$=\frac{1}{2}[c_{fL}(z,\tilde{\bar{U}}_j)S_{wL}(z)+\tilde{c}_d(z,\tilde{\bar{U}}_j)S_{wT}(z)+c_{dp}(z,\tilde{\bar{U}}_j)S_{wP}(z)]\tilde{\bar{U}}_j^2$$

$$+\frac{1}{\rho_f}\frac{d\langle\bar{p}_j\rangle_f}{dx}=c_dS_w\frac{\tilde{\bar{U}}_j^2}{2}+\frac{1}{\rho_f}\frac{d\langle\bar{p}_j\rangle_f}{dx}, \qquad (450)$$

where

$$\tilde{K}_{mj}=\tilde{K}_{mj}+\nu_j,$$

and the lumped flow resistance coefficient c_d is the complex morphology dependent function. The energy equation in the jth fluid phase is

$$c_{pfj}\rho_{fj}\langle m\rangle\tilde{\bar{U}}_j(z)\frac{\partial\tilde{T}_j(x,z)}{\partial x}=\frac{\partial}{\partial z}(\langle m(z)\rangle\tilde{K}_{Tj}(z)\frac{\partial\tilde{\bar{T}}_j(x,z)}{\partial z}$$

$$+\tilde{\alpha}_T(z)S_w(z)(T_s(x,z)-\tilde{T}_j(x,z)), \qquad (451)$$

with $(x, z) \in \Delta\Omega_f$, and the energy equation in the solid phase

$$\frac{\partial}{\partial z}(\langle 1 - m(z)\rangle K_{ST}(z) \frac{\partial T_s(x, z)}{\partial z}$$
$$= \tilde{\alpha}_T(z) S_w(z)(T_s(x, z) - \tilde{T}_j(x, z))(x, z) \in \Delta\Omega, \tag{452}$$

with

$$P_{rT} \approx 1 : \tilde{K}_{Tj} \approx \tilde{K}_{mj} c_{pfj} \rho_{fj} + k_{fj}, \tag{453}$$

where index j determines the fluid phase number $j = 1, 2$ in conjugate channels 1 and 2.

In Eqs. (444), (445), (450), and (452), the coefficient functions and specific surface functions must be determined by assuming real or invented morphological models of the porous structure. The pressure gradient term in Eq. (450) is modeled as a constant value in the layer, or simulated by the local value of the right-hand side of the experimental correlations. The boundary conditions for these equations are

$$z = 0 : \tilde{\tilde{U}}_j = 0, \quad \frac{\partial b_j}{\partial z} = 0$$

$$\tilde{K}_m = v, \quad Q_0 = -\tilde{K}_{Tj} \frac{\partial \tilde{T}_j}{\partial z}$$

$$Q_0 = -K_{ST} \frac{\partial T_S}{\partial z} \tag{454}$$

$$z = \frac{+}{-} h_j : \frac{\partial \tilde{\tilde{U}}_j}{\partial z} = 0, \quad \frac{\partial b_j}{\partial z} = 0$$

$$\frac{\partial \tilde{T}_j}{\partial z} = 0, \quad \frac{\partial T_S}{\partial z} = 0, \tag{455}$$

where h_j is the half channel width. The control terms in the preceding equations depend on temperature and velocity distributions as well as on morphological characteristics of the media.

Comparing the three latest equation (450)–(452) with the equations derived by Paffenbarger [206] for practically the same structural design of HE, one will find numerous discrepancies. For example, the energy balance equations in Paffenbarger's [206] work have energy conservation terms that do not match each other.

The VAT-based general transport equations for a single phase fluid in an HE medium have more integral and differential terms than the homogenized or classical continuum mechanics equations. Various descriptions of the porous medium structural morphology determines the importance of these terms and the range of application of the closure schemes. Prescribing regular, assigned, or statistical structure to the capillary or globular HE medium morphology gives the basis for transforming the integrodifferential transport equations into differential equations with probability density functions governing their stochastic coefficients and source terms. Several different closure models for these terms for some uniform, nonuniform, nonisotropic, and specifically random nonisotropic highly porous layers were developed in work by Travkin and Catton [16, 17, 23], etc. The natural way to close the integral terms in the transfer equations is to attempt to find the integrals over the interphase surface, or over outlined areas of this surface. Closure models allow one to find connections between experimental correlations for bulk processes and the simulation representation and then incorporate them into numerical procedures.

D. Optimal Control Problems in Heat Exchanger Design

A variety of the optimization problems that can be formulated in the area of heterogeneous medium transport involve differential equations modeling the physics of the process. Many of them have a fairly complicated form. The contemporary literature on optimal control deals with problems that are mathematically similar but consider much simpler formulations of the optimization problem with constraints in the form of differential equations. Linear optimal control systems governed by parabolic partial differential equations (PPDEs) are relatively well studied. The CHE modeling equations resulting from the VAT-based analysis are also PPDEs, but they are nonlinear and have additional integral and integrodifferential terms. The models presented and the resulting differential equations contain additional integral and integrodifferential terms not studied in the literature.

The performance of a heat exchanger depends on the design criteria for optimizing the liquid flow velocity, dimensions of the heat exchanger, the heat transfer area between hot side and cold side, etc. Thermal optimization of an HE requires selection of many features—for example, both the optimum fin spacing and optimum fin thickness, each determined to maximize total heat dissipation for a given added mass or profile area. These criteria set the optimal conditions for HE operation. Theoretically, the optimal dimensions of an HE require a large number of tiny tubelets with diameters tending to zero with increasing number of tubes. This leads to a very fine dispersion problem with porous medium–like behavior. Extremely compact micro heat exchangers with plate–fin cross flow have already been built. However, the optimization problems involving such designs are more complex than traditional designs and require new simulation techniques.

E. A VAT-Based Optimization Technique for Heat Exchangers

A variety of optimal control problems that can be formulated in the area of heterogeneous medium transport involve differential equations modeling the physics of the process. Some of them have a fairly complicated form. Meanwhile, the contemporary literature on optimal control considers too simple formulations of the optimization problems with constraints in form of differential equations.

Optimal control systems governed by parabolic partial differential equations have been studied intensively. For example, Ahmed and Teo [214] give a survey on main results in this field. Questions concerning necessary conditions for optimality and existence of optimal controls for these problems have been investigated in work by Ahmed and Teo (215–217] and Fleming [218]. Moreover, a few results by Teo *et al.* (1980) on the computational methods of finding optimal controls are also available in the literature (Teo and Wu [213]). However, turbulent transport equations in highly porous media were proposed by Travkin *et al.* [19] for optimization problems and developed in more detail in Section IV with additional "morphlogical" as well as integral and integrodifferential terms. Recent literature studies show optimal control problems involving PPDE either in general form or in divergence form and propose computational methods such as variational technique and gradient method (see, for example, Ahmed and Teo [214]). These studies seems to be helpful for solving various optimization problems involving integro–differential transport equations considered by Travkin *et al.* [19]. However, complete research has to be done for this class of equations, including analysis of necessary conditions and existence of optimal control, as well as developing computational methods for solving various optimal control problems.

Optimal control for some classes of integrodifferential equations has also been considered in recent years. Da Prato and Ichikawa [219] studied the quadratic control problems for integrodifferential equations of parabolic type. A state-space representation of the system is obtained by choosing an appropriate product space. By using the standard method based on the Riccati equation, a unique optimal control over a finite horizon and under a stabilizability condition is obtained and the quadratic problem over an infinite horizon is solved. Butkovski [220] was the first to discuss the optimal control problems for distributed parameter systems. The maximum principle as a set of necessary conditions for optimal control of distributed parameter systems has been studied by many authors.

Since it is well known that the maximum principle may be false for distributed parameter systems (see Balakrishnan [221]), there are many papers that give some conditions ensuring that the maximum principle remains true (see, for example, Ahmed and Teo [214]; Balakrishnan [221]; Curtain and Pritchard [222]). We note that the references just mentioned discuss the cases for distributed parameter systems or functional differential systems with no end constraints and/or with the control domain being convex; thus, they do not include Pontryagin's original result on maximum principle as a special case.

Fattorini [223] also proposed an existence theory and formulated maximum principle for relaxed infinite-dimensional optimal control problems. He considered relaxed optimal control problems described by semilinear systems ODE and used relaxed controls whose values are finitely additive probability measures. Under suitable conditions, relaxed trajectories coincide with those obtained from differential inclusions. The existence theorems for relaxed controls were obtained; they are applied to distributed parameter systems described by semilinear parabolic and wave equations, as well as a version of Pontryagin's maximum principle for relaxed optimal control problems.

Optimal control problems involving equations such as (432)–(438) have control terms with the structures

$$\nabla(\langle m\rangle\{f_1(\bar{x})\nabla f_2(\bar{x})\}_f)$$

$$\nabla(\langle m\rangle\{f_3(\bar{x})f_2(\bar{x})\}_f)$$

$$\nabla\left(\varphi_1(\bar{x})\int_{\partial S_w} f_2(\bar{x})\cdot\vec{d}s\right)$$

$$\int_{\partial S_w}[\varphi_2(\bar{x})\nabla(f_4(\bar{x}, f_2(\bar{x})))]\cdot\vec{d}s, \qquad (456)$$

with controls f_1, f_2, f_3, f_4. Such statements of the control problem are hardly seen in the contemporary literature on optimal control distributed-parameter systems (see, for example, Ahmed and Teo [214]). The existence of optimal controls for equations much simpler than those here were developed only very recently; see Fattorini [223]. Thus, for linear heat- and mass-diffusion problems with impulse control that is a function of magnitude or spatial locations of the impulses, Anita [224] obtained a formulation of maximum principles for both optimal problems. Ahmed and Xiang [225] proved the existence of optimal controls for clear nonlinear evolution equations on Banach spaces with the control term in the equations being represented as an additive–multiplicative term $B(t)u(t)$.

Reduction of "hererogeneous" terms in the corresponding momentum equation by an overall representation of diffusive and "diffusionlike" terms yields

$$k_{m,eff} \frac{\partial \tilde{\bar{A}}}{\partial x} = \left[\langle m \rangle (\tilde{K}_m + \nu) \frac{\partial \tilde{\bar{A}}}{\partial x} + \left\langle \tilde{K}_m \frac{\partial \hat{a}}{\partial x} \right\rangle_f + \langle -\hat{a}\hat{a} \rangle_f \right]. \tag{457}$$

Here, the velocity and fluctuating viscosity coefficient variables are taken in a form suitable for both laminar and turbulent flow regimes. For problems with a constant bulk viscosity coefficient ($K_m = \text{constant}$), the second term in this relation vanishes and the whole problem essentially becomes one of evaluating the influence of dispersion by irregularities of the soil medium on the momentum. Thermal dispersion effects realized through the second derivative terms and relaxation terms and, for example, in the fluid phase with constant thermal characteristics heat transport dispersion can be expressed as

$$K_{T,eff} \frac{\partial \tilde{\bar{T}}}{\partial x} = \left[\langle m \rangle (\tilde{K}_T + k_f) \frac{\partial \tilde{\bar{T}}}{\partial x} + \left\langle \hat{K}_T \frac{\partial \hat{\bar{T}}}{\partial x} \right\rangle_f - c_{pf} \rho_f \langle m \rangle \{ \tilde{\bar{T}} \hat{u} \}_f + \frac{(\tilde{K}_T + k_f)}{\Delta\Omega} \int_{\partial S_w} \tilde{\bar{T}} \vec{d}s \right], \tag{458}$$

where the first and last terms resemble the effective thermal conductivity coefficient for each phase, using constant coefficients, found in the work by Nozad *et al.* [40]. By allowing the control terms to be added to the bulk transport coefficients, another variation of a mathematical statement for optimal control can be found.

As far as optimal control problems with PDE dynamics are concerned, one can find a detailed solution of the linear quadratic regulator problem, including conditions for the convergence of modal approximation schemes. However, for more general optimal control problems involving PDE, the main approach has been to use some method for constructing a particular finite-dimensional approximating optimal control problem and then to solve this problem. The relationship between the solutions and stationary points of the approximating optimal control problem and those of the original optimal control problem is not established in these papers.

For the models and differential equations describing HEs to be useful, the additional integral and integrodifferential terms need to be addressed in a systematic way. VAT has the unique ability to enable the combination of direct general physical and mathematical problem statement analysis with the convenience of the segmented analysis usually employed in HE design. A segmented approach is a method where overall physical processes or groups of phenomena are divided into selected subprocesses or phenomena that are interconnected to others by an adopted chain or set of dependencies. A few of the obvious steps that need to be taken are the following:

1. Model what increases the heat transfer rate
2. Model what decreases of flow resistance (pressure drop)
3. Combine the transport (thermal/mass transfer) analysis and structural analysis (spatial) and design
4. Find the minimum volume (the combination of parameters yielding a minimum weight HE)
5. Include nonlinear conditions and nonlinear physical characteristics into analysis and design procedures

The power and convenience of this method is clear, but its credibility is greatly undermined by variability and freedom of choice in selection of subportions of the whole system or process. The greatest weakness is that the whole process of phenomena described by a voluntarily assigned set of rules for the description of each segment is sometimes done without serious consideration of the implications of such segmentation. Strict physical analysis and consideration of the consequences of segmentation is not possible without a strict formulation of the problem that the VAT-based modeling supplies. Structural optimization of a plate HE, for example, using the VAT approach might consist of the following steps: (1) optimization of the number of plates, plate spacing and fin spacing; (2) optimization of the fin shape; (3) simultaneous optimization of multiple mathematical statements. This approach also allows consideration and description of hydraulically and thermally developing processes by representing them through the distributed partial differential systems.

X. New Optimization Technique for Material Design Based on VAT

A variety of optimal control problems that can be formulated in the area of heterogeneous medium transport involve differential equations modeling the physics of the process. Many of them have a fairly complicated form, and the contemporary literature on optimal control considers much simpler formulations of the optimization problems with constraints in form of differential equations.

When the diffusion equations are written in nonlocal VAT form, there are additional terms appearing in the mathematical statements. These terms can be considered to be morphology controls involving differential and integral operators. The nonlinear diffusion equation written without source terms has three control terms,

$$\langle s_1\rangle \frac{\partial C_1}{\partial t} = \nabla\cdot(\tilde{D}_1\nabla\langle s_1\rangle\tilde{C}_1) + \nabla\cdot\left[\tilde{D}_1 \frac{1}{\Delta\Omega}\int_{\partial S_{1\beta}} C_1 \vec{d}s_1\right]$$
$$+ \nabla\cdot(\langle\hat{D}_1\nabla\hat{c}_1\rangle) + \frac{1}{\Delta\Omega}\int_{\partial S_{1\beta}} D_1\nabla C_1\cdot\vec{d}s_1$$
$$= \nabla\cdot(\tilde{D}_1\nabla\langle s_1\rangle\tilde{C}_1) + F_{C1}(C_1, \tilde{D}_1, M_\omega) + F_{C2}(\hat{c}_1, \hat{D}_1, M_\omega)$$
$$+ F_{C3}(C_1, D_1, M_\omega), \tag{459}$$

where the morphology characteristics set M_ω contains many parameters, ω_n, such as phase fraction $\langle s_1\rangle$ and specific surface area ∂S_{12},

$$M_\omega \subset (\langle s_1\rangle, \partial S_{1\beta}, \omega_3, \omega_3, \ldots).$$

The equation for an electrostatic electrical field in a particulate medium (polycrystalline medium) is

$$\nabla\cdot[\langle s_1\rangle\tilde{\varepsilon}_1\tilde{\mathbf{E}}_1] + \nabla\cdot\langle\hat{\varepsilon}_1\hat{\mathbf{E}}_1\rangle_1 + \frac{1}{\Delta\Omega}\int_{\partial S_{12}} (\varepsilon_1\mathbf{E}_1)\cdot\vec{d}s_1 = \langle\rho\rangle_1,$$

which becomes

$$\nabla\cdot[\langle s_1\rangle\tilde{\varepsilon}_1\tilde{\mathbf{E}}_1] + F_{E1}(\hat{\varepsilon}_1, \hat{\mathbf{E}}_1, M_\omega) + F_{E2}(\varepsilon_1, \mathbf{E}_1, M_\omega) = \langle\rho\rangle_1. \tag{460}$$

Additional equations are

$$\nabla\times(\langle s_1\rangle\tilde{\mathbf{E}}_1) + \frac{1}{\Delta\Omega}\int_{\partial S_{12}} \vec{d}s_1\times\mathbf{E}_1 = 0$$
$$\nabla\times(\langle s_1\rangle\tilde{\mathbf{E}}_1) + F_{E3}(\mathbf{E}_1, M_\omega) = 0. \tag{461}$$

A temperature control equation for the solid phase with the two morphology control terms can be written

$$\frac{\partial\tilde{T}_m}{\partial t} = a_m\frac{\partial^2\tilde{T}_m}{\partial z^2} + T_{mMin}(T_m, \partial S_{12}, t, z) + T_{mMqin}(T_m, \partial S_w, t, z), \tag{462}$$

where

$$T_{mMin} = a_m\frac{\partial}{\partial z}\left[\frac{1}{\Delta\Omega_m}\int_{\partial S_{12}} T_m\vec{d}s_1\right], \quad T_{mMqin} = \frac{a_m}{\Delta\Omega_m}\int_{\partial S_{12}} \frac{\partial T_m}{\partial x_i}\cdot\vec{d}s_1, \tag{463}$$

and in the void phase

$$\frac{\partial\{T_2\}_2}{\partial t} = a_2\frac{\partial^2\{T_2\}_2}{\partial z^2} + T_{2Min}(T_2, t, z) + T_{2Mqin}(T_2, t, z)$$

$$T_{2Min}(T_2, \partial S_{12}, t, z) = a_2\frac{\partial}{\partial z}\left[\frac{1}{\Delta\Omega_2}\int_{\partial S_{12}} T_2\vec{d}s_2\right] \tag{464}$$

$$T_{2Mqin}(T_2, \partial S_{12}, t, z) = \frac{a_2}{\Delta\Omega_2}\int_{\partial S_{12}} \nabla T_2\cdot\vec{d}s_2. \tag{465}$$

These terms are not equal and their calculation or estimation presents a challenge. However, these are the real driving forces that will differentiate the behavior of one composite from another. Their application will lead to a direct connection between design goals and morphological solutions.

XI. Concluding Remarks

Determination of the effective parameters in model equations are usually based on a medium morphology model and there are dozens of associated quasi-homogeneous and quasi-stochastic methods that claim to accomplish this. In most cases, quasi-homogeneous and quasi-stochastic methods have no well treated solutions and, most important, they are not sufficient for description of the physical process features in heterogeneous media, especially when treating a multiscale processes.

The hierarchical approach applied to radiative transfer in a porous medium and to the electrodynamics governing equations (Maxwell's equations) in a heterogeneous medium yielded new volume averaged radiative transfer equations—VAREs. These equations have additional terms reflecting the influence of interfaces and inhomogeneities on radiation intensity in a porous medium and, when solved, will allow one to relate the lower scale parameters to the upper scale material behavior. The general nature of this result makes it applicable to any subatomic particle transport, including neutron transport, as well as radiative transport in the heterogeneous media field. Direct closure based on theoretical and numerical developments that have been developed for thermal, momentum, and mass transport processes in a specific random porous and composite medium established a basis for closure modeling in problems in radiative and electromagnetic phenomena.

In this work, transport models and equation sets were obtained for a number of different circumstances with a well substantiated mathematical theory called volume averaging theory (VAT) that included linear, nonlinear, laminar, and turbulent hierarchical transport in nonisotropic heterogeneous media, accounting for modeling level, interphase exchange, and microroughness. Models were developed, for example, for porous media using an advanced averaging technique, a hierarchical modeling methodology, and fully turbulent models with Reynolds stresses and fluxes. It is worth noting that nonlocal mathematical modeling is very different from homogenization modeling. The new integrodifferential transport statements in heterogeneous media and application of these nonclassical types of equations is the current issue. The theory allows one to take into consideration characteristics of multicomponent multiphase composites with perfect as well as imperfect morphologies and interphases. The transport equations obtained using VAT involved additional terms that quantify the influence of the medium morphology. Various descriptions of the porous medium structural morphology determine the importance of these terms and the range of application of closure schemes.

Many mathematical models currently in use have not received a critical review because there was nothing to review them against. The more common models were compared with the more rigorous VAT-based models and found deficient in many respects. This does not mean they do not serve a useful purpose. Rather, they are incomplete and suffer from lack of generality.

VAT-based modeling is very powerful, allowing random morphology fluctuations to be incorporated into the VAT-based transport equations by means of randomly varying morphoconvective and morphodiffusive terms. Closure of some of the resulting morphofluctuation in the governing transport equations has been outlined, resulting in some well-developed closure expressions for the VAT-based transport equations in porous media. Some of them exploit the properties of available solutions to transport problems for individual morphological elements, and others are based on the natural morphological data of porous media.

Statistical and numerical techniques were applied to classical irregular morphologies to treat the morphodiffusive and morphoconvective terms along with integral terms. The challenging problem in irregular and random morphologies is to produce an analytical or numerical evaluation of the deviations in scalar or vector fields. In previous work, the authors have presented a few exact closures for predetermined regular and random porous medium morphologies. The questions related to effective coefficient dependencies, boundary conditions, and porous medium experiment analysis are discussed.

Analysis of heat exchanger designs depends on the heat balance equations that are widely used in the heat design industry. A theoretical basis for employing heat and momentum transport equations obtained with volume averaging theory was developed for modeling and design of heat exchangers. This application of VAT results in a correct set of mathematical equations for heat exchanger modeling and optimization through implementation of general field equations rather than the usual balance equations. The

performance of a heat exchanger depends on the design criteria for optimizing the liquid flow velocity, dimensions of the heat exchanger, the heat transfer area between the hot side and cold side, etc. However, the optimization problems involving such designs are more complex than for traditional designs and require new optimal control simulation techniques.

A variety of optimal control problems that can be formulated in the area of heterogeneous medium transport involve differential equations modeling the physics of the process. Many of them have a fairly complicated form, and the contemporary literature on optimal control considers much simpler formulations of the optimization problems with constraints in the form of differential equations. Linear optimal control systems governed by parabolic partial differential equations (PDEs) are relatively well studied in the literature. The modeling CHE equations resulting from VAT-based analysis are also PDEs, but they are nonlinear and have additional integral and integrodifferential terms.

It is well known that some matrix composites (often porous) represent the promise for design of a series of materials with highly desirable characteristics such as high temperature accommodation and enhanced toughness. Their performance is very dependent on the volume fraction of the constituent materials, reinforcement interface and matrix morphologies, and consolidation. Scale characteristics (nanostructural composites) give the abnormal physical properties, such as magnetic, and mechanical transport and state a great challenge in formulating the hierarchical models containing the design objectives.

The importance of the physical processes taking place in a heterogeneous multiscale–multiphase–composite medium creates the need for the development of new tools to characterize such media. It leads to the development of new approaches to describing these processes. One of them (VAT) has great advantages and is the subject of this review.

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Nomenclature

a	thermal diffusivity [m^2/s]	$\bar{c}_d$	mean skin friction coefficient over the turbulent area of ∂S_w[-]
c_d	mean drag resistance coefficient in the REV [-]		

c_{dp}	mean form resistance coefficient in the REV [-]
$c_{d,sph}$	drag resistance coefficient upon single sphere [-]
c_{fL}	mean skin friction coefficient over the laminar region inside of the REV [-]
c_p	specific heat [J/(kg·K)]
C_1	constant coefficient in Kolmogorov turbulent exchange coefficient correlation [-]
d_{ch}	character pore size in the cross section [m]
d_i	diameter of *i*th pore [m]
d_p	particle diameter [m]
ds	interphase differential area in porous medium [m²]
D_f	molecular diffusion coefficient [m²/s]; also tube or pore diameter [m]
D_h	flat channel hydraulic diameter [m]
D_s	diffusion coefficient in solid [m²/s]
∂S_w	internal surface in the REV [m²]
$\tilde{f} \equiv \{f\}_f$	averaged over $\Delta\Omega_f$ value f—*intrinsic averaged variable*
$\langle f \rangle_f$	value f, averaged over $\Delta\Omega_f$ din an REV—phase averaged variable
$\hat{f}$	morphofluctuation value of f in a Ω_f
g	gravitational constant [1/m²]
H	width of the channel [m]
h	averaged heat transfer coefficient over ∂S_w [W/(m²/K)]; half-width of the channel [m]
h_r	pore scale microroughness layer thickness [m]
∂S_w	internal surface in the REV [m²]
k_f	fluid thermal conductivity [W/(mK)]
k_s	solid phase thermal conductivity [W/(mK)]
K	permeability [m²]
K_b	turbulent kinetic energy exchange coefficient [m²/s]
K_c	turbulent diffusion coefficient [m²/s]
K_m	turbulent eddy viscosity [m²/s]
K_{sT}	effective thermal conductivity of solid phase [W/(mK)]
K_T	turbulent eddy thermal conductivity [W/(mK)]
l	turbulence mixing length [m]
L	scale [m]
$\langle m \rangle$	averaged porosity [-]
m_s	surface porosity [-]
n	number of pores [-]
n_i	number of pores with diameter of type i [-]
Nu_{por}	$=\dfrac{\bar{h}_s d_h}{\lambda_f}$, interface surface Nusselt number [-]
p	pressure [Pa]; or pitch in regular porous 2D and 3D medium [m]; or phase function [-]
Pe_h	$=Re_h Pr$, Darcy velocity pore scale Peclet number [-]
Pe_p	$=Re_v Pr$, particle radius Peclet number [-]
Pr	$=\dfrac{\nu}{a_f}$, Prandtl number [-]
Q_0	outward heat flux [W/m²]
Re_{ch}	Reynolds number of pore hydraulic diameter [-]
Re_h	$=\dfrac{\langle m \rangle \tilde{\bar{u}} d_h}{\nu}$, Darcy velocity Reynolds number of pore hydraulic diameter [-]
Re_p	$=\dfrac{\tilde{\bar{u}} d_p}{\nu}$, particle Reynolds number [-]
Re_{por}	$=\dfrac{\tilde{\bar{u}} d_{por}}{\nu}$, Reynolds number of general scale pore hydraulic diameter [-]
S_{cr}	total cross-sectional area available to flow [m²]
S_w	specific surface of a porous medium $\partial S_w/\Delta\Omega$ [1/m]
S_{wp}	$=S_\perp/\Delta\Omega$ [1/m]

$S_\perp$	$=S_{pr}$ cross flow projected area of obstacles [m²]
T	temperature [K]
T_a	characteristic temperature for given temperature range [K]
T_s	solid phase temperature [K]
T_w	wall temperature [K]
T_0	reference temperature [K]
U, u	velocity in x direction [m/s]
u_{rk}^2	square friction velocity at the upper boundary of HR averaged over surface ∂S_w [m²/s²]
V	velocity [m/s]
V_D	$=\tilde{\bar{u}}\langle m \rangle$ Darcy velocity [m/s]
W	velocity in z direction [m/s]

SUBSCRIPTS

e	effective
f	fluid phase
i	component of turbulent vector variable; or species or pore type
k	component of turbulent variable that designates turbulent "microeffects" on a pore level
L	laminar
m	scale value or medium
r	roughness
s	solid phase
T	turbulent
w	wall

SUPERSCRIPTS

~	value in fluid phase averaged over the REV
^	value in solid phase averaged over the REV
¯	mean turbulent quantity
′	turbulent fluctuation value
*	equilibrium values at the assigned surface or complex conjugate variable

GREEK LETTERS

$\bar{\alpha}_T$	averaged heat transfer coefficient over ∂S_w [W/(m²K)]
$\Delta\Omega$	representative elementary volume (REV) [m³]
$\Delta\Omega_f$	pore volume in a REV [m³]
$\Delta\Omega_s$	solid phase volume in a REV [m³]
ε_d, ε_m	electric permittivity [Fr/m]
μ	dynamic viscosity [kg/(ms)] or [Pas]
μ_m	magnetic permeability [H/m]
ν	kinematic viscosity [m²/s]; also ν, frequency [Hz]
ρ	density [kg/m³]; also ρ, electric charge density [C/m³]
σ_e	medium specific electric conductivity [A/V/m]
Φ	electric scalar potential [V]
ψ	particle intensity per unit energy (frequency)
$\{\psi\}$	ensemble-averaged value of ψ
$\tilde{\psi}_j^s$	interface ensemble-averaged value of ψ, with phase j being to the left
ω	angular frequency [rad/s]
χ	magnetic susceptibility [-]
$\kappa_{va} = \kappa_a$	absorption coefficient [1/m]
$\kappa_{vs} = \kappa_s$	scattering coefficient [1/m]

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